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THE RESPONSE OF THE HUMAN SKULL TO MECHANICAL VIBRATIONS

ERNST K. FRANKE

AERO MEDICAL LABORATORY

NOVEMBER 1954

Statement A
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THE RESPONSE OF THE HUMAN SKULL TO MECHANICAL VIBRATIONS

ERNST K. FRANKE

AERO MEDICAL LABORATORY

NOVEMBER 1954

PROJECT No. 7210

TASK No. 71704

**WRIGHT AIR DEVELOPMENT CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO**

FOREWORD

This research was conducted by the Bio-Acoustics Section of the Aero Medical Laboratory, Directorate of Research, Wright Air Development Center, under the authority of Project No. 7210, Human Response to Vibratory Energy, Task No. 71704, The Response of the Human Body to External Mechanical Vibrations. Dr. Ernst K. Franke served as the project scientist.

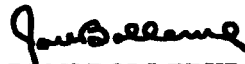
Part of the work described in Section III of this report was carried out at the Department of Anatomy, College of Medicine, Ohio State University, with the kind permission of Dean R. C. Baker. The help of Prof. Ralph Stacy and Dr. Alfred Fasola is gratefully acknowledged.

ABSTRACT

This report describes measurements of the mechanical impedance and of the resonance frequencies of the human skull. The measurements were made in the frequency range from 200 to 1,600 cps, the skull being excited to vibration by means of an electrodynamically driven piston with a small contact area. Data were obtained from living subjects, a dry skull preparation and a human cadaver. The modulus of elasticity of skull bone, calculated from the resonance frequency of the skull, is consistent with the value obtained by static measuring methods. The propagation velocity of bending waves in the skull bones, also calculated from the resonance frequency, agrees satisfactorily with the experimentally determined propagation velocity. It is shown, finally, that a vibrating spherical shell is a suitable model for the skull and describes its vibration patterns with good approximation.

PUBLICATION REVIEW

This report has been reviewed and is approved.
FOR THE COMMANDER:



JACK BOLLERUD
Colonel, USAF (MC)
Chief, Aero Medical Laboratory
Directorate of Research

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SECTION I INTRODUCTION

The human skull, when exposed to mechanical vibration either by contact with vibrating structures or by action of a sound field, will respond by vibrating in certain modes with an intensity depending on its own structural peculiarities and the parameters of the external stimulus. Investigation and explanation of its response are problems in their own right, the solution of which will contribute to the general understanding of body mechanics. Beyond this basic interest, the knowledge of the various aspects of skull vibration will further the development of the theories of hearing by bone conduction and of side tone propagation. It may also bear upon the response of the head to impulsive forces which produce skull fracture or brain concussion. In spite of its importance, knowledge of the mechanical properties of the human skull is still limited. Moreover, there are contradictions between the results of different experimenters. For this reason it appeared worth while to work on problems of skull vibration.

Attempts to obtain experimental data for the skull have, thus far, met certain difficulties which are commonly found in experiments with human subjects. For instance, measurements of the human mastoid (1), which were made previously in this laboratory to obtain design data for an artificial mastoid, did not show any appreciable influence on skull vibration. Instead, the experiment revealed that the mechanical properties of the surface of the head are controlled exclusively by the soft layer of skin overlying the skull bones. This difficulty was avoided by using a dry skull preparation in a tentative experiment that was performed with the intent to determine the fundamental resonance frequency of the skull. In such a preparation, a strong coupling between the measurement device and the skull is easily obtained. The measurement revealed that the skull has a resonance at about 800 cps; in living subjects, its frequency may well be expected to be considerably lower because of the strong damping effect of the attached tissues. This value of resonance frequency, however, is at variance with data for living subjects previously published in the literature. G. von Békésy (2) for instance, has suggested 1600 cps; in a later review of this subject (3) however, he has stated without further explanation, that the skull resonance for living subjects is about 800 cps. This is still higher than our own preliminary experiments would indicate. Barany (4), in his extensive treatise on hearing by bone conduction, unfortunately used a frequency range too limited to have any bearing on the problem of skull resonance and skull vibration in general. For all these reasons it was decided to re-investigate the mechanical response of the skull to vibration and to establish a sufficiently simple mechanical model which describes the essential features of the experiment. The results of these investigations are presented and discussed in the following Sections.

SECTION II

IMPEDANCE MEASUREMENTS ON A DRY HUMAN SKULL PREPARATION

Two main reasons make the measurements on the living skull particularly difficult. First, the human skull is filled with brain, a highly viscous fluid* strongly damping the vibrations on the head.

* In a mechanical sense, the brain tissue may be considered as a viscous fluid in the frequency range of the wave motion considered in this report.

Secondly, the skull is lined on its outer side with rather pliable tissue which prevents sufficiently strong coupling between mechanical measuring devices and the bone. Both obstacles can be removed by using a dry human skull preparation. In this case, however, it must be determined whether the resonance frequencies of respective modes of vibration are at least approximately the same as those in living subjects. To obtain an estimate, one must first find out how much the elastic modulus of the bone is changed by the drying process. According to F. Gaynor Evans and M. Lebow (5), the elastic modulus of a dry bone is, on the average, about 20% higher than that of a wet bone. Since the resonance frequency of any geometric configuration depends upon the square root of E, the difference between the living skull and this dry preparation will be in the order of 10%. This means that the inaccuracy of resonance determination will, very likely, be greater than the inaccuracy due to drying.

The second objection to the use of a dry preparation is the possible effect of tissue viscosity on the resonance. It may be expected that the damping is very high in a living subject, even so high that the displacement is not appreciably enhanced at resonance. It is for this reason that previous attempts to measure the resonance frequency by recording the displacement amplitude of a living skull as a function of frequency may have been unsuccessful. Therefore, one has good reason to believe that the damping will be almost aperiodic. The coefficient of damping would then be approximately 0.5. If this assumption -- which is probably an overestimate -- were correct, the fundamental natural frequency of a dry skull preparation would be about 25% greater than that of a living subject. This figure means that the measurement of the resonance frequency of a dry skull preparation will give information at least comparable as to the order of magnitude with that of living subjects.

The following measurement method was used: The occipital side of the human skull preparation was put on a small, rigid support and its frontal side brought in contact with a vibrator. The tip of the vibrator contained a pressure sensitive element which recorded the amplitude and phase of the vibratory force exerted by the driver on the skull. In addition, the amplitude and phase of the velocity of the piston of the vibrator was measured. The impedance of the skull was then calculated and the natural frequencies obtained from the locus of impedance in the complex plane. For details of the method of measurement and its theory, the reader is referred to the work of Franke (6). Two improvements on the method described in this reference have since been made and may be worth mentioning. One of them is the use of a drive magnet which is less bulky and easier to manipulate. The other is the change from the measurement of the displacement of the piston to the measurement of its acceleration. The motion of the piston must be measured in an inertial system. This condition is difficult to fulfill by measuring displacement. However, by recording the acceleration of the piston, it is accomplished automatically.

At first, the skull preparation was used without any attempt to simulate the damping effect of the brain fluid. The impedance locus thus obtained is shown in Figure 1. On the abscissa is plotted the resistance and on the ordinate, the reactance. The frequency is shown as a parameter. The natural frequency of the system can be read at the point where $X=0$ and $R = R_{max}$. In this case it is approximately 820 cps. The general shape of the locus curve shows that the equivalent electrical network is a parallel resonant circuit with the resonance impedance

$$I = \frac{ms}{r} = 830 \times 10^4 \text{ dynes/CM/sec}$$

where m is the mass of an equivalent system of one degree of freedom (considered to be concentrated to a point-mass), s its stiffness and r its resistance. It is not possible to separate the equivalent circuit elements with the aid of the information obtained so far. The mechanical properties of the living skull may be assumed to be strongly influenced by the properties of the attached tissues. Two phenomena may be expected; one is the decrease in resonance amplitude, the other, the reduction of resonance frequency. Therefore, an experiment was conducted with the skull filled with gelatin to obtain at least an approximation of the actual viscosity of the human brain (Appendix I). The results of the measurements are shown in Figure 2. In this diagram, the

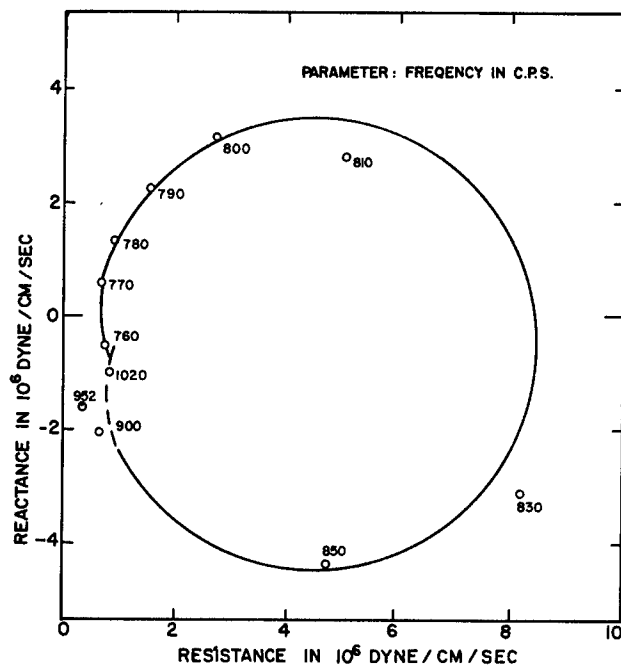


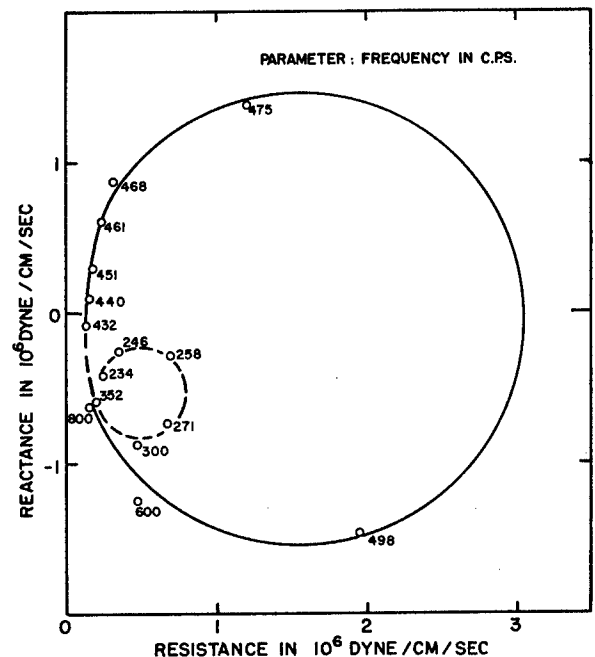
Figure 1. The mechanical impedance locus of a dry skull preparation.

the impedance locus has the same shape, corresponding to a vibrator of one degree of freedom. Its details, however, are different from those obtained with an air-filled skull. The resonance impedance has decreased to

$$I_{nes} = 300 \times 10^4 \text{ dyne/cm/sec.}$$

The resonance frequency is reduced to approximately 500 cps. This is in accordance with the theoretical considerations in the first paragraphs of this Section. In Section IV, the theoretical part of this report, the mode of vibration which corresponds to this resonance frequency will be discussed.

Figure 2. The mechanical impedance locus of a gelatin filled dry skull preparation.



SECTION III

IMPEDANCE MEASUREMENTS ON HUMAN SUBJECTS

Using the same method as described in Section II, the impedance of the heads of living human subjects and in one instance, of a cadaver, were measured. Figure 3 shows a typical sample of the results of measurement of the forehead impedance of a human subject. The curves are almost identical with those found over the mastoid (1). During the experiment the subject sat upright on a chair. The occiput was loosely supported and the vibrations were applied to the forehead by a horizontally vibrating piston. The support of the occiput had no influence on the results of the measurements in the frequency range under investigation. As previously stated (1), no indication of skull resonance or bone vibration could be found. The reactance measured is virtually that of a soft spring with a resonance that is almost constant. Therefore, one may conclude that it is the reactance of the layer of tissues overlying the hard skull. The coupling between the driving piston and the skull is so loose that no measurable reactions of the vibrations of the skull to the piston are detectable.

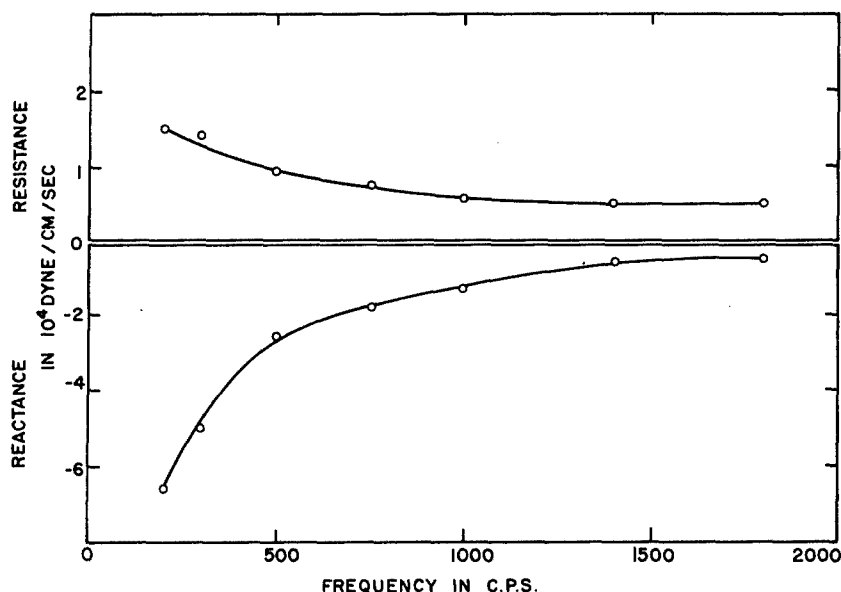


Figure 3. Mechanical resistance and reactance measured on the forehead of a living human subject.

To obtain better coupling between the piston and the skull, it was decided to measure the impedance of the head of a human cadaver after removal of the skin and periosteum. Figure 4 shows the reactance and the resistance measured after removal of the skin; the periosteum is still in place. The general shape of the curve is the same as that in Figure 3, that is, the piston of the measuring device is still acting against a predominantly elastic reactance.

But there are also significant differences. The absolute value of elastic resistance is many times higher than that measured with the skin still in place. There is also a slight trace of resonance of the whole skull structure. By carefully looking at Figure 4, one can observe a deviation from the $1/\omega$ frequency response at approximately 600 to 1000 cps; this fact indicates a small deviation from

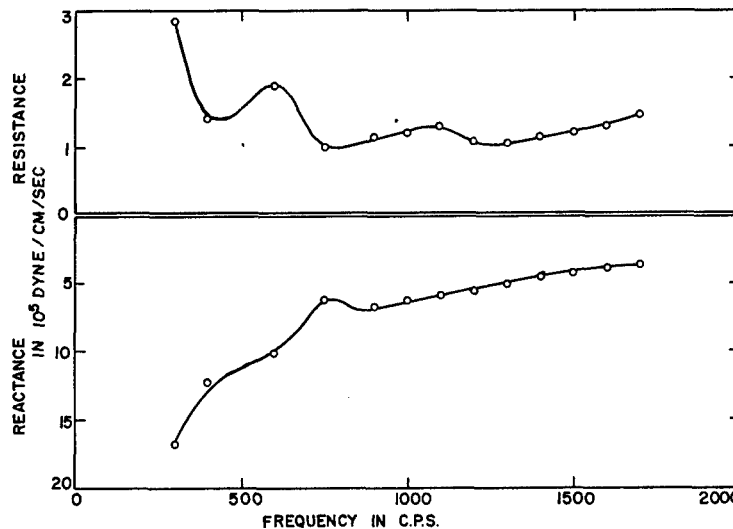


Figure 4. Mechanical resistance and reactance measured on the forehead of a human cadaver after removal of the skin.

the predominantly elastic character of the reactance. At the same frequencies, the resistance also deviates from the regular behavior. The change will stand out more clearly when the curves of Figure 4 are combined into one graph of the impedance locus (Figure 5) with the frequency as a parameter. There is evidence of an extremely damped resonance at 600 cps, and another slightly less damped resonance at approximately 900 cps. The accuracy of the measurement is rather small because of the high stiffness and comparably small resistance (6). The same results are obtained after removal of the periosteum. In the following Section it will be seen that these frequencies are very close to the resonance frequency that is observed on the living skull by means of other methods. Quantitative evaluation of the curves of Figures 4 and 5 similar to the manner used for Figures 1 and 2 is not possible because of the inherently low accuracy of measurement.

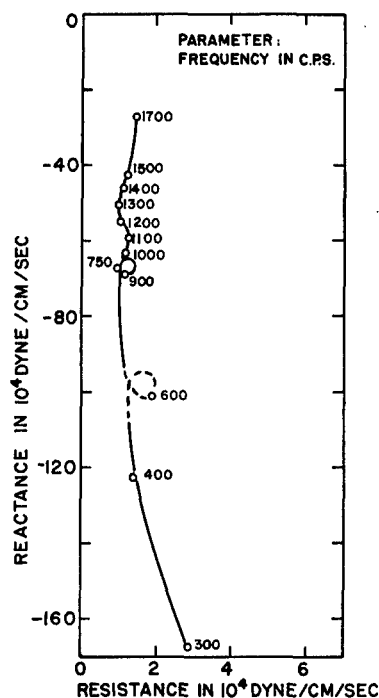


Figure 5. The mechanical locus of the forehead of a human cadaver after removal of the skin.

SECTION IV

THE MEASUREMENT OF NODAL LINES OF VIBRATION ON THE HEADS OF HUMAN SUBJECTS

The modes of skull vibration may be determined by measuring the relative phase of motion at different points of the head of a subject although, as shown in Section III, the resonance amplitude is too faint to give any useful clue. This method was first employed by von Békésy (2). He obtained, for the lowest excitable mode, a frequency of 1600 cps. This result is considerably at variance with the data in Section II of this report which show that the frequency of the modes, even of a dry preparation, are well below 1000 cps. In a more recent publication (3) von Békésy mentions 800 cps as the resonance frequency of the head without giving experimental details. Since the knowledge of the resonance frequency of the head is of theoretical as well as practical interest, for instance in bone conduction hearing, the measurements of von Békésy were repeated in a somewhat modified form.

The following method was used; The head was excited to vibrations by means of an electro-dynamically actuated piston. The design of the piston which was pressed against the middle of the forehead was similar to that used for the impedance measurements of Sections II and III. The reference voltage for the determination of the phase was obtained by pressing a bone conduction transducer (used as a pickup) against the middle of the occiput. The surface of the head was then probed with another pickup; (for the necessary conditions which a pickup should fulfill to be an accurate measuring instrument, see von Bekesy (2)). The phase difference of the two pickups was read on a commercial audio-frequency phase meter and plotted as shown in Figures 6 to 8.

In passing, it may be mentioned that the driving voltage of the piston cannot be used as a phase reference since changes in attachment pressure which are difficult to avoid in experiments involving living subjects, will change the relative phase between piston and head. In fact, the phase fluctuations were so great that measuring was virtually impossible. A reference pickup placed near the piston also was tried but could not be used because, in this case, the vibrations recorded were caused by two different mechanisms. One is the vibration of the underlying bone, the vibration to be measured; the other is a strongly damped shear wave propagated along the tissues covering the bone. The latter also depends so much on attachment pressure that stable recording is impossible. With the reference pickup at the occiput, however, very satisfactory reading of the phase was finally obtained since the relative phase between the two pickups now depends only on the vibration of the underlying bone. Any phase change due to fluctuations of the attachment pressure is transmitted equally to both transducers and cancels out in their relative phase.

Curve a of Figure 6 shows the phase at various points on the circumference of the head at a frequency of 300 cps. The phase reverses approximately in the middle transverse line of the head. It will be explained in Section VI that this is the characteristic of the first mode of vibration of a spherical shell. This mode, in the present case, is essentially a displacement of the skull as a whole, parallel to itself. In Curve b of Figure 6 at 400 cps, another line of phase reversal has appeared near the point of excitation at the forehead. On the other hand, the node line corresponding to the first node has moved a little toward the occiput. This behavior is well in accordance with the theoretical properties of a closed spherical shell (see Section VI) where the node lines change their position as a function of frequency during transition from one mode to the next higher one. In Figure 7 at 600 and 750 cps, a second node line is already well established near the forehead and the first has moved still farther backward. Figures 6 and 7 demonstrate that the resonance frequency of the second mode of the skull - which is, incidentally, the lowest bending mode that can be excited here as well as in the experimental arrangements of Sections II and III - falls in the range between 400 and 600 cps. This frequency is in the order of magnitude of the corresponding frequency of the gelatin filled preparation (580 cps) and very likely somewhat lower. This result is not surprising because of the fact that the damping of the head of a living subject is much greater than that of the preparation.

At 900 cps (Curve a of Figure 8) the second mode still prevails and is about symmetrically developed. Curve b of Figure 8 shows the situation at 1200 cps. Here the third node line has appeared while the other node lines have again moved farther to the left. The transition point from the second to the third mode is now somewhere between 900 and 1200 cps. The exact frequency is hard to determine because the transition is gradual in contrast to the undamped sphere where the transition takes place in a rather short frequency interval. The theoretical frequency ratio between the third and second mode is 1.5 (see Section VI). The frequencies of the second and third modes which are compatible with this figure as well as the results of the measurements, are 500 and 850 cps. These figures are well in agreement with the results obtained by the different methods discussed in Sections II and III, and show that the resonance frequency of the skull is considerably lower than previously assumed.

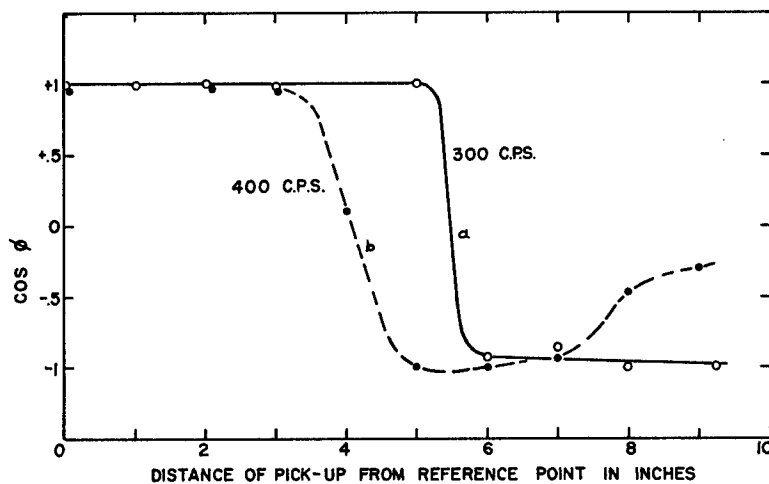


Figure 6. Phase of vibration on the circumference of the head at 300 cps and 400 cps.

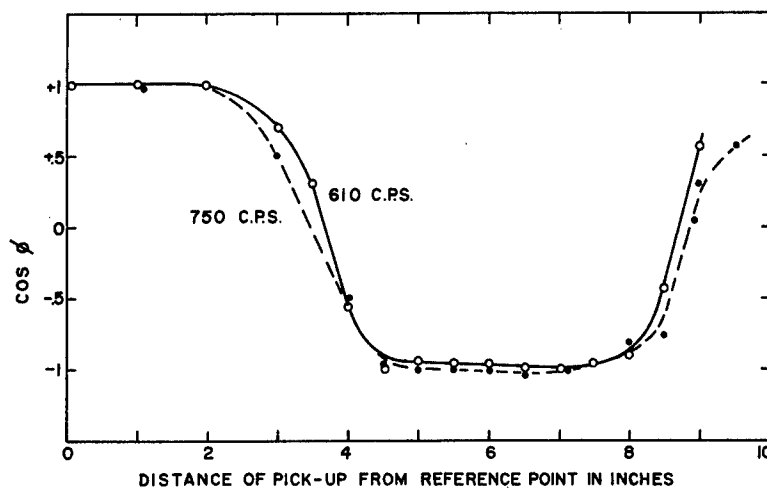


Figure 7. Phase of vibration on the circumference of the head at 600 cps and 750 cps.

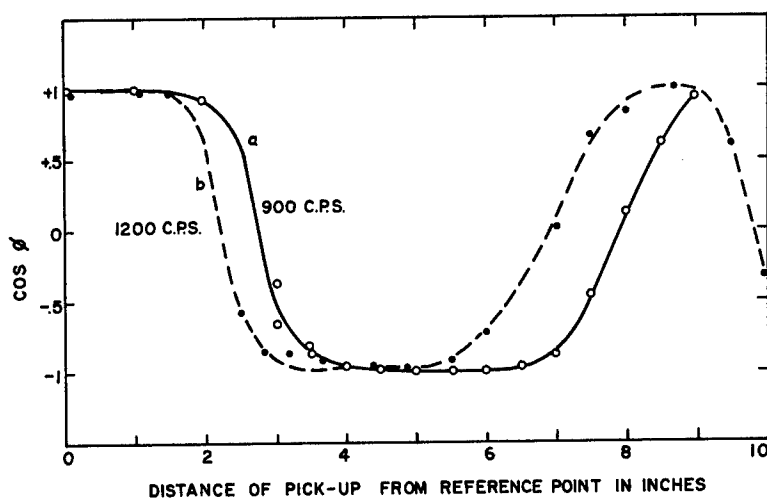


Figure 8. Phase of vibration on the circumference of the head at 900 cps and 1200 cps.

SECTION V

THE VELOCITY OF PROPAGATION OF SKULL VIBRATION

The velocity of propagation of the vibration of the skull may be derived from the frequency of the second mode. Because it can also be measured directly without theoretical assumptions, it may be used conveniently as a means of checking the validity of the results presented in the preceding sections. The frequency of the second mode is approximately 500 cps, and the full wave length is one half the circumference of the skull. In the subject used for the determination of the modes, the circumference is about 25 cm after due allowance for skin and periosteum. From these figures a wave velocity of 125 m/sec is computed.

In contrast to this figure, von Békésy (2) had derived a much higher velocity, i.e., 540 m/sec, from his resonance data. He also tried to verify his calculations by measuring the difference in time of arrival of a signal at the forehead and occiput. As a signal, he used clicking of the incisors. As the path length, he chose the full distance between the midpoints of forehead and occiput, where he had placed the pickups. Some objections may be made, however, to the choice of path. It seems that the time difference between the two paths, i.e., incisor-forehead and incisor-occiput, is smaller than the time that the wave would need to travel directly from forehead to occiput. Von Békésy (2) argued theoretically that most of the vibratory energy that eventually reaches the occiput will travel over the forehead. But this is an assumption which would have required experimental verification and is also at variance with the data obtained in experiments with "side tone" propagation.

There is also another point which is open to criticism, the choice of a click as a signal. A click has essentially a continuous spectrum; the spectrum of an actual signal arriving at the forehead and occiput is shaped by the mechanical properties of the skull which act as a mechanical filter. Even then, it covers a wide band of frequencies, and different frequencies may travel with different velocities. It has been found, indeed, that in the case of the propagation velocity of surface waves over muscle tissue there is considerable dispersion of velocity (7). The dispersion (frequency dependence of velocity) is caused by the presence of viscosity in the tissues. Because of the dispersion, the determination of the velocity is different since different frequencies arrive at different times. It will be virtually impossible to determine which frequency arrives first.

For all these reasons, it appeared necessary to make a new attempt to determine the velocity of propagation, the more so because this experiment is crucial to the validity of the results presented in the other portions of this report. The methods and apparatus were designed to avoid the deficiencies commented on in the foregoing paragraph. First, the signal was not generated by clicking of the teeth but by means of an electro-magnetically driven piston similar to that used for the impedance measurements. In this way, one is free to choose the site of excitation in the most satisfactory manner. The point of excitation, therefore, was put at the forehead as near as practically possible, to a pickup placed on the forehead. The second pickup was placed near the occiput and connected to the first by means of adhesive tape (loosely stretched to avoid transmission of vibrations) to keep a fixed distance of 12 or 6 cm, respectively. Because the first pickup is so near the piston, it is certain that there is no faster path between the forehead and the occiput.

To avoid the difficulties inherent in the broad spectrum of the click, a short wave train was used consisting of five full periods of sinusoidal shape with a slowly rising and falling amplitude. The envelope of the whole pulse train was chosen according to a cosine function. The pulse was repetitive but the time between two consecutive pulses was long enough to allow the first pulse to die out entirely before the next arrived. The envelope of the pulse spectrum is shown in Figure 9. Its frequency range is sufficiently narrow to permit specifying a predominant frequency and to avoid virtually any appreciable dispersion of the components of the spectrum during transmission.

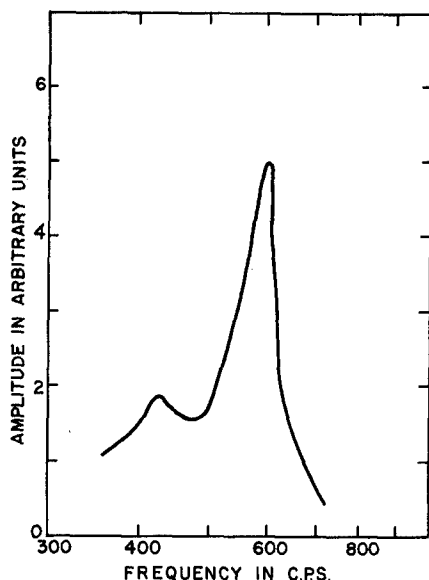


Figure 9. Envelope of the spectrum of the wave train used to determine the propagation velocity of mechanical vibrations in skull bones.

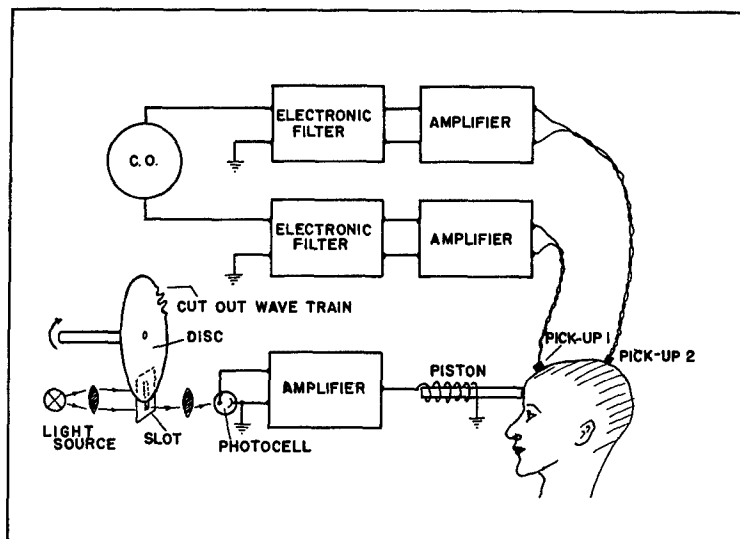


Figure 10. Block diagram of the apparatus for the measurement of the propagation velocity of mechanical vibration in skull bones.

A block diagram of the measuring apparatus is shown in Figure 10. The pulse wave train is generated by means of an electro-optical device consisting of a rotating disk made of paper or cardboard, the pulse train being cut out on an appropriate part of the circumference. The disk rotates in front of an illuminated slot, partially interrupting the light incident to a photocell. The electrical signal from the photocell is amplified and transmitted to the driver. The pickups are placed on the head of the subject. Their electrical outputs are then amplified, filtered and connected to the input terminals of a dual beam oscilloscope. Both channels are as nearly identical as possible to avoid phase shift between the two signals generated in the pickups. In spite of this precaution, the phase of the signals was not equal when the signal was applied to both pickups simultaneously for a check. In the actual experiment, therefore, records were taken twice for each frequency, the place of the pickups being exchanged for the second reading. The time difference in the arrival of the signal was then calculated from the two corresponding traces recorded on the oscilloscope. In this way, the phase difference between the channels was cancelled. For convenience, the sweep of the oscilloscope was synchronized with the rotating disk. The recording was done with a Polaroid camera. Figure 11 shows a sample record; the frequency is 1000 cps and the time delay calculated to be 0.4 milliseconds.

The results of measurements at different frequencies are plotted in Figure 12. It is apparent that a considerable dispersion exists. At the lowest frequency measurable, the velocity of propagation is about 80 m/sec. With the frequency increasing, the velocity rises slowly at first, but between 500 and 1000 cps, at a much faster rate and approximately proportional to the frequency. Above 1000 cps, it levels off at about 300 m/sec. The velocity near 500 cps is about 150 m/sec, well in accordance with the value of 125 m/sec calculated in the preceding Section. The measurements of the resonance frequency, therefore, are well supported by the results of the determination of the propagation velocity.

For the dispersion of velocity, only a qualitative explanation can be offered at this time. Accurate measurements of the propagation velocity of surface waves over the thigh (7) showed a strikingly similar behavior. This can be explained by the tissue being visco-elastic. The evaluation of the

equation of wave motion in a semi-infinite visco-elastic medium (8) gives a velocity of propagation which depends on frequency in about the same manner as in the experiments on the skull. By analogy, therefore, it appears likely that also in the case of propagation in bony tissues, the presence of viscosity will account for the existence of the dispersion.

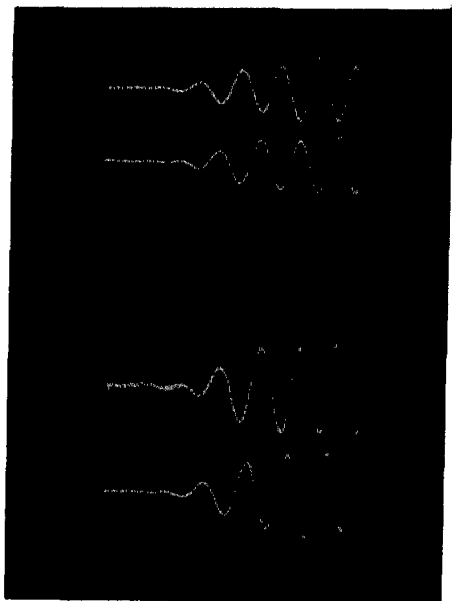


Figure 11. Sample record of a wave train.

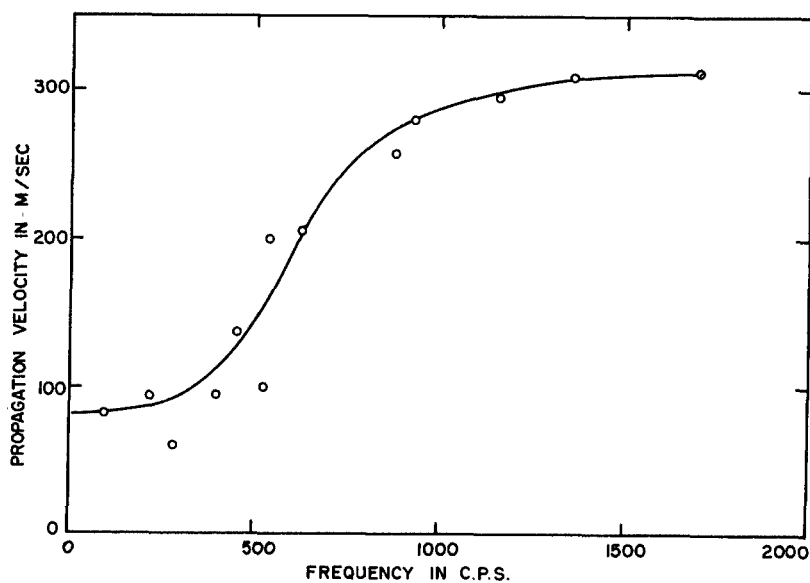


Figure 12. Propagation velocity of vibrations in skull bones as a function of frequency.

SECTION VI

COMPARISON OF THE VIBRATING SKULL WITH A VIBRATING CLOSED SPHERICAL SHELL

To get an approximate theoretical model of the vibration patterns of the skull, an attempt will be made to compare its vibratory properties with those of a closed, thin-walled spherical shell. This is a rather crude approximation, of course, primarily because the head is not uniform. The facial bone attached to the brain capsule and the cervical bones, for instance, will certainly cause considerable deviations from the behavior of the simplified model. On the other hand, the sphere is the only body geometrically related to the skull that has been mathematically treated. A theory of the vibrations of the oblong rotation ellipsoid, the next better approximation, is not yet available. Any configuration giving a still better approximation will probably present insurmountable mathematical difficulties. The vibration of a closed spherical shell has been treated by Love (9). A further development of his results and, at the same time, a more modern presentation, has been worked out by Junger (10). He gives, on page 443, c.f., an equation of the mathematical impedance of a spherical shell vibrating in vacuo

$$Z_n = j \frac{2h}{\omega} \left\{ \rho \omega^2 - \frac{E}{(1-\gamma)a^2} \left[2 - \frac{1+\gamma}{1 - \frac{1-\gamma}{n(n+1)} \left[\frac{\omega^2 \rho (1+\gamma)a^2 + 1}{E} \right]} \right] \right\} \quad (1)$$

for the node n , where

ω = angular frequency
 h = half-thickness of shell
 ρ = density of shell
 E = Young's modulus
 γ = Poisson's ratio
 n = integer (number of mode)
 $\delta = \sqrt{1-\gamma}$
 a = radius of shell

Since the superposition of modes is possible, he obtains for the radial component of the dynamic displacement

$$f = f \frac{e^{j\omega t}}{4j\pi a^2 \omega} \sum_{n=0}^{\infty} \frac{2n+1}{Z_n} P_n(\cos \theta) \quad (2)$$

These equations hold in case of a radially directed harmonic force $f \exp(j\omega t)$ applied at the pole $\theta = 0$ of a sphere. $P_n(\cos \theta)$ are Legendre's polynomials. Since the transverse component of displacement cannot be measured with the methods described in previous parts of this paper, it will be omitted from the following considerations. For the discussion, it is advantageous to express in dimensionless form that part of the equation 1 which depends on frequency (term in braces). For this purpose the angular frequency of the lowest mode ω_0 , is introduced which is found by putting $n=0$ and it follows

$$\rho \omega_0^2 = \frac{2E}{(1-\gamma)a^2} \quad \text{or} \quad \omega_0^2 = \frac{2E}{\rho(1-\gamma)a^2} \quad (3)$$

For the impedance, we get then

$$Z_n = 2j h \left[\rho \omega - \frac{2E}{(1-\gamma)a^2 \omega} + \frac{(1+\gamma) P_n \omega_0^2 / 2}{\omega - \frac{1-\gamma}{n(n+1)} \left[\frac{\omega^3}{\omega_0^2} \cdot 2 + \omega \right]} \right] \quad (4)$$

Now a dimensionless normalized frequency, η , is defined by putting $\omega/\omega_0 = \eta$ and hence

$$Z_\eta = 2jh \left[\rho \omega_0 \eta - \frac{\omega_0}{\eta} + \frac{\rho}{\eta} \frac{(1+r)\omega_0}{2 \left[1 - \frac{1-r}{\eta(\eta+1)} (2\eta^2 + 1) \right]} \right] \quad (5)$$

After some elementary calculation it follows;

$$Z_\eta = 2jh\omega_0 \rho \left[\eta - \frac{1}{\eta} + \frac{1}{\eta} \frac{1+r}{2 \left[1 - \frac{1-r}{\eta(\eta+1)} (2\eta^2 + 1) \right]} \right] \quad (6)$$

It can now be seen that the dimensions are contained in the constant term $2jh\omega_0\rho$ whereas the variable part, i.e., the function of a normalized frequency, η , is dimensionless. This part is, therefore, independent of thickness and radius of the skull. Equation 5 permits the calculation of the ratios of the frequencies of the higher modes to the frequency of the lowest mode:

$$\begin{aligned} \eta_0 &= 1 \quad \text{as assumed} \\ \eta_1 &= \sqrt{\frac{5}{6}} \quad \text{that is, the frequency of the first mode is} \\ &\quad \text{slightly lower than that of the zero mode.} \\ \eta_2 &= 1.87 \\ \eta_3 &= 2.74 \\ \eta_4 &= 3.51 \end{aligned}$$

The resonance frequencies have been determined in such a way as to render the impedance zero for $\eta = 0, 1, 2, 3, 4$, etc. That means that one may, for every independent mode, represent the vibrating sphere by an equivalent network of an inductance and a capacitance connected in a series, with negligible damping.

Through the combined action of the modes, the sphere also has frequencies where it presents an infinite impedance to the driving piston, as one may see from the respective numerical calculation. Since these frequencies are rather close to the frequencies where the impedance is low, it will suffice to consider only the frequencies where $Z_\eta = 0$.

The different modes will now be considered separately. In the zero mode, the radial displacement of the sphere is independent of the meridian. This is the case of the pulsating sphere. It is very unlikely that this mode can be excited by either unsymmetrical (piston on forehead, head approximately free) or symmetrical force (piston on forehead, occiput supported). It may be possible, however, to excite this mode in a sound field. For the first mode, $\eta = 1$, we have a node meridian at $\theta = 90^\circ$. This mode can be excited by unsymmetrical forces only. Since, in experimental arrangement only phases are measured, this case is undistinguishable from the case of ordinary translation of the sphere parallel to itself, since the latter also gives a node at $\theta = 90^\circ$ for the radial component.

The second mode, with a node meridian at both $\theta=55^\circ$ and $\theta=125^\circ$ can be excited in both experimental arrangements. Therefore, it is most important for our case. In the preceding Section where the details of the measurement have been discussed, it was made rather certain that it is this mode that appears at approximately 500 or 600 cps in the measurements of impedance of the dry skull preparation as well as in the phase measurements on the head of the subject.

The third mode, $\theta=40^\circ, 90^\circ, 140^\circ$, is unsymmetrical again and was found only in the phase measurements. A trace of it may also be present in the impedance of the skull of a human cadaver; in this case, it may have been superimposed on the second mode vibrations because the head is not very rigidly supported at the occiput because of the tissues.

The fourth mode occurs at frequencies higher than those used in the experiment.

It was shown in Section IV that the node meridian moves slowly away from the point of excitation (forehead) with rising frequency. Such motion of nodes of displacement is well known on membranes and it also takes place in shells. The node shift during the transition from the first to the second mode has been calculated as a function of η . The results will be described at first qualitatively; at $\eta = 1.8$, the first mode prevails, the node meridian lies between 80° and 90° (not exactly

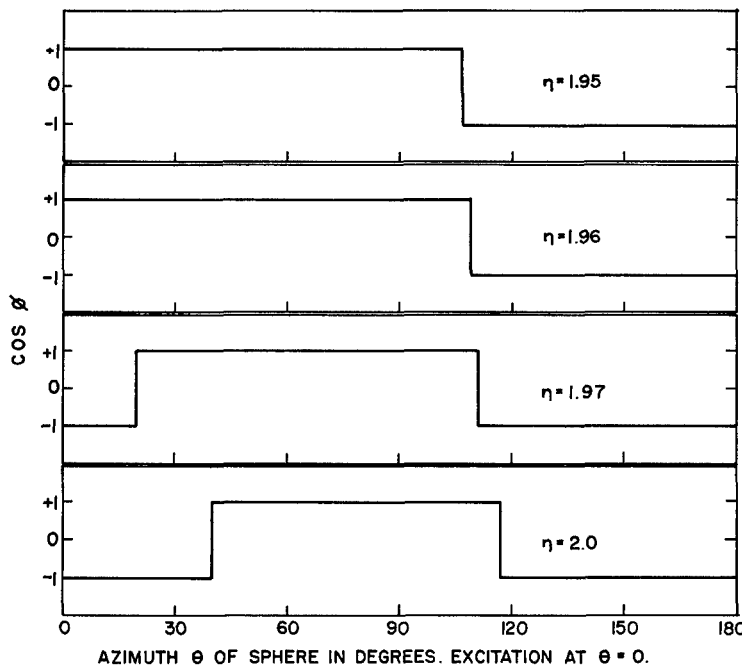


Figure 13. Phase of vibration on the circumference of a closed spherical shell.

for the pure second mode. From now on, with the frequency rising further, position of the nodes remains fairly stable until the appearance of the third node. Figure 13 represents the phase of motion with respect to the exciting force for different normalized frequencies. The similarity with Figures 6 to 8 is at once apparent. This confirms the validity of the sphere as a model for theoretical studies of skull vibration patterns.

at 90° because there is already a slight influence of the second mode). At $\eta = 1.95$, the node has moved to $\theta = 105^\circ$. From there on, it moves very rapidly, and at $\eta = 1.96$, the second mode appears with a node at $\theta = 0^\circ$, at the point of excitation. This also means that at this frequency the impedance is infinite. The previous node meridian has moved to approximately 110° . $\eta = 1.96$ is only slightly higher than $\eta = 1.87$, which is the frequency of the impedance minimum. This is a quantitative verification of our previous statement that the frequencies where, in the undamped case, $Z_\eta = 0$ and $Z_\eta = \infty$ are rather close together. With damping, the frequency difference may be somewhat greater. In this case, the change of position of nodes with frequency may also be slightly lower. At $\eta = 1.97$ there are now two well established node meridians at $\theta = 20^\circ$ and $\theta = 110^\circ$. At $\eta = 2.0$ the meridians have moved further but not so rapidly as before and are now at $\theta = 40^\circ$ and $\theta = 120^\circ$, not far from their theoretical position

APPENDIX I

THE VISCOSITY OF BRAIN TISSUE

Oestreicher (8) has worked out the theory of a vibrating rigid sphere in a viscous elastic infinite medium. Specifically, he has worked out the driving point impedance of the sphere as a function of the properties of the medium. Working backward, the elasticity and viscosity constants of the medium can be calculated by application of this theory after the mechanical impedance of the vibrating sphere has been measured. In this case, the sphere is a first order source. Applying the theory to brain tissue, one is justified to make some simplifying assumptions. First, the attenuation of the waves is, fortunately, sufficiently high so that the condition of infinite medium is easily satisfied. In other words, the reaction of the walls of a rigid vessel containing tissue will be negligibly small, even when the container is not very large. A cylindrical vessel of approximately 25 cm diameter and a filling height of about 15 cm was used. The frequency range extended to 500 cps only. This means that shear waves are present. Compression waves would start at frequencies well above the range of interest for the present report. According to reference (4), we have the resistance of the sphere

$$R = 6 \pi a^2 \sqrt{\frac{\rho}{\mu_2}} \sqrt{\frac{\omega}{2}} + 6 \pi a \mu_2$$

where

a = radius of the sphere
 ρ = density of medium
 ω = angular frequency
 μ_2 = coefficient of shear viscosity

With a sphere of the diameter of $2a = 2 \text{ cm}$ the value

$$\mu_2 \approx 20 \quad \text{c.g.s. units of viscosity}$$

was found from the measured resistance and the above equation.

Methods:

The container was first filled with whole pig brains to the extent stated above. The sphere (blown glass sphere attached to a thin glass tube) was immersed in approximately the center of the material and attached to the same impedance measuring apparatus that was used for the measurements described in Sections II and III. The brain was very fresh, used not later than two hours after slaughtering of the animals, and kept at body temperature all the time. The frequencies of measurement were 150, 125 and 500 cps, the upper frequency limit given by the inherent compliance of the glass rod. To obtain an estimate of the influence on the results of the fact that whole brains were used and that, therefore, the mass was not quite homogeneous, contrary to the premises of the theory, the brains were ground afterwards and the measurements repeated. There was no significant difference.

The value of viscosity is in the order of magnitude of that of glycerin at room temperature, and much smaller than the viscosity found for the other tissues.

APPENDIX II

THE ELASTICITY OF THE SKULL BONES

Equation 3 (page 11 this report) permits the computation of Young's modulus, E , by introducing numerical values for the radius of the skull and the zero mode resonance frequency. The latter is not directly observable but can be calculated from the observable resonance frequency of the second mode (about 800 cps for the dry preparation) and the theoretical ratio $\eta_2/\eta_0 = 1.81$. Hence the elastic modulus of the dry skull preparation is

$$E = 1.4 \times 10^{10} \text{ dynes/cm}^2$$

The value available in the literature(5, 11) for the elasticity of bone is

$$E = 1.0 \times 10^{11} \text{ dynes/cm}^2$$

This value was measured on samples taken from different sites on the femur. In particular, the samples were cut from the hard layers of the bone only and did not contain any spongy material. The elastic modulus determining the resonance of the skull, however, is determined by the whole bone, including the spongy part. Therefore, it seemed worthwhile to make a few static measurements of the elastic modulus of the whole bone of the skull used in the experiments. Test samples were cut from the skull preparation at three sites, frontal bone, above the temporal and parietal bone. Modulus was determined by bending the samples. The accuracy of the static tests was not too great, primarily because the thickness of the samples was rather irregular. Unfortunately, shaping of the samples would remove materially, the hard outer layer only, thus significantly changing the elasticity of the sample. The results for E were 3.5×10^{10} ; 2.0×10^{10} and 0.8×10^{10} dynes/cm², respectively. This is exactly the same order of magnitude as computed from the resonance.

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